Markov Chain Analysis of Manpower data of a Khutul Cement Lime company

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Abstract

According to the source, since 2010 Mongolia’s construction sector has had at least 5 percent annual growth, and its growth has been leading in comparison to other sectors. The increased growth in the construction industry led to increased demand in construction materials, especially consumption of cement has increased dramatically. Due to Mongolian government’s initiative to expand production of construction material in order to supply construction materials domestically, projects of technologically advanced cement industry were financed with 108.3 million dollars through government bonds. This is a great incentive for the development of cement production industry. Along with this a problem of providing the required professional workforce arises. This sector confronts many obstacles including seasonal stoppage and operation, dependency from import, shortage of workforce. Furthermore, it suffers from the fact that most of the financing came from short-term high-interest bank loans. As long as this sector creates large amount of temporary jobs, it goes into a high-risk sector for human resource according to the definition of International Labor Organization. The study shows that 66.7 percent of Khutul Cement and Lime Plant workers are professionals, and they hire temporary staffs during peak season. Lower educated employees are often hired due to unreasonable absence at work or for failing to follow job responsibilities. The risk of losing its professional experienced staffs to competitor companies arose due to new technologically advanced cement manufacturing plants. The need for studying workforce flow and movement is justified by the reasons stated above.

Keywords: Markov chain analysis, transition probability matrix (TPM), absorbing states, valued digraph

INTRODUCTION

The approach to manpower policy in most Khutul Cement – Lime industry appears to be guided by the traditional method of putting the right number of people in the right place at the right time or arranging for suitable number of people to be allocated to various jobs usually in a hierarchical structure. The technique is outdated because it lags behind the state-of-the-art method that deals with manpower policy in the context of organizational strategy. The traditional method is deficit in the sense that it neither offers computational tools that will enable managers to determine possible line of action to be taken to steer manpower policy to desired ends nor provide tools to generate alternative policies and strategies.

The method advocated is exploratory and through its computational tools, can generate outcomes that will enable normative models to be formulated. In this regard, prescriptive standard that can guide manpower policy to the desired direction can be easily established. The traditional methods are naive and therefore lack this potency.

Manpower planning modelling has been accomplished by different approaches but the Markov Chain approach appears to offer more intuitive appeal than others like optimization method such as simulation, renewal theory, decision calculus, among others. Markov chain modelling is one of the most powerful tools for analyzing complex stochastic systems (Kim and Smith, 1989). The versatility of Markov chain is widely acclaimed. Slatyer (1977) is an excellent material that discussed a wide range of possible applications of Markov Chain.

Markov chain has also been applied successively to population genetics (Bartholomew and Forbes, 1976). Moreover, Anthony and Taylor (1977) explored the use of Markov models in forecasting air pollution levels. The analysis of historical data concerning variations in air pollution indices suggests a pattern which might usefully be described by a transition probability matrix. Zanakis and Marret (1980) noted that personnel supply in an organization can be forecasted using Markov Chains to model the flow of people through various states (usually skill or position levels). Such applications are treated in Rowland and Sovereign (1969), Nielson and Young (1973) and Merch (1970). While manpower flow through minority status is reported in Churchill and Shanks (1975), flow based on years of service is studied by Merch (1970), Leeson (1979). Parthasarathy et al (2010) adopted stochastic models to manpower planning in an organization and showed that for a two grade manpower system, exponentiated exponential distribution can be used to determine when the cumulative loss of manpower crosses a random threshold level that is detrimental to

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Markov chain models have become popular in manpower system planning. Several researchers have adopted Markov chain models to clarify manpower policy issues: Kim and Smith (1989), Heyman and Sobel (1982), Zanakis and Maret (1980), Calantone and Darmon (1984).

In summary, one can say that there have been several attempts at applying Markov chain to a wide range of problem situations. A myriad of these situations exist and attempts at applying Markov chain to a wide range of problems have been recorded. Thus, the probability of an edge upon which a transition occurs being a Markov chain model, the probability distribution of the current state is a Markov chain model. The characteristic called the Markov property. Markov chain is a discrete-time stochastic process with the Markov property. The probability that a sequence x is generated by a Markov chain model.

$P(x) = P(x_1, x_2, ..., x_n) = P(x_1, x_2, ..., x_{n-1}) \cdot P(x_n | x_1, x_2, ..., x_{n-1})$

By applying many times of

$P(X, Y) = P(X) \cdot P(Y | X)$

One assumption of Markov chain is that the probability of $x_t$ only depend on the previous symbol $x_{t-1}$, i.e.,

$P(x_n | x_1, x_2, ..., x_{n-1}) = P(x_n | x_{n-1})$

Thus

$P(x) = P(x_1) \cdot P(x_2 | x_1) \cdot P(x_3 | x_2) \cdot ... \cdot P(x_n | x_{n-1})$

In this model, we must specify the probability $P(x_1)$ as well as the transition probabilities $a_{x_t, x_{t+1}}$. To make the formula homogeneous (i.e., comprise of only terms in the form of $a_{x_t, x_{t+1}}$), we can introduce a begin state to the model. Figure 1 depicts the valued diagraph of a Markov Chain transition among states space, $S = \{S_1, S_2, ..., S_6\}$.

Figure 1 Valued diagram

This is both basic and applied research in the sense that the paper offers theoretical knowledge upon which application to solve fundamental human resources planning problems in the organization studied is established. The structure and strategy of our investigation comprise description of the manpower structure of the company being studied. For mathematical tractability, the states space investigated was condensed from sixteen states to mere six states which are enumerated as follows: (i) Disciplinary case (Dc) (ii) Staff Stock (S) (iii) Leave (L) (iv) Contract (C) (v) Wastage (W) (vi) Retirement (R).

By states, it is specifically implied the condition, the status, the position or situation a Markov Chain (object or staff) undergoes in the transition process towards the final position of retirement, if the staff can ever get there. The staff movement, in steps, in this system is considered to be a non-cyclic ergodic chain that appears regular. A forty year data of staff movements, as specified by state space described above, was obtained from the organization studied. The population of staff is 4,381. Basic assumptions were explicitly stated and applicable theorems and formulae governing the computation were presented.

A purposive and quota sampling methods were used. Markov Chain model was used to analyse the data collected. A valued diagram (Markov Transition Diagram) was developed to enable one discern the structure of transition probability matrix. Finally, the data obtained were used to compute the transition probability matrix (TPM) which we depict in Table 1. The estimates of the transition probabilities were based...
on frequency distributions or tabulations of the number of transitions from one state to the other in the system considered. The frequency was converted to TPM by dividing each row by its total.

Table 1 Lamped states space (2011-2014)

<table>
<thead>
<tr>
<th>States</th>
<th>Retirement</th>
<th>Wages</th>
<th>Disciplinary case</th>
<th>Leave</th>
<th>Staff stock</th>
<th>Contract</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manpower</td>
<td>7</td>
<td>119</td>
<td>69</td>
<td>71</td>
<td>3758</td>
<td>367</td>
<td>4381</td>
</tr>
</tbody>
</table>

RESULTS

The theory governing movement of personnel among well-defined states (i.e., status, conditions, and positions) in the organisation studied is hereby briefly presented. Record of staff movement over a period of 5 years was obtained from a typical Khutul Cement Lime company and the Markov Chain model was fitted into the record. Consider a staff recruited and who begins transition from initial distribution do among the six states. The long run distribution is expressed as:

\[ T = d_0 T_n \]  

(1)

Where \( T = \) stabilized transition, in this case \( n = 5 \) years (\( T_n \)). It is assumed that after this number of movements, the matrix \( T \) would have attained stochastic regularity (stationarity).

\[ T = \begin{bmatrix} I + O \end{bmatrix} \frac{R}{Q} \]

\[ T = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \frac{0.01827}{0.01078} \]

\[ d_o = R \begin{bmatrix} 0.01827 \ 0.01078 \ 0 \end{bmatrix} \]

\[ Q = \begin{bmatrix} 0.00109 & 0.85980 & 0.10043 \ 0.01109 & 0.87484 & 0.1022 \ 0.0101 & 0.86806 & 0.1014 \end{bmatrix} \]

From this initial distribution \( d_0 \) if it transits to other states as follows:

\[ T^2 = \begin{bmatrix} I + O \end{bmatrix} \frac{R}{Q} \]

Thus

\[ d_o = R \begin{bmatrix} 0.01827 \ 0.01078 \ 0 \end{bmatrix} \]

\[ Q = \begin{bmatrix} 0.00109 & 0.85980 & 0.10043 \ 0.01109 & 0.87484 & 0.1022 \ 0.0101 & 0.86806 & 0.1014 \end{bmatrix} \]

From this initial distribution \( d_0 \) it transits to other states as follows:

\[ T^3 = \begin{bmatrix} I + O \end{bmatrix} \frac{R}{Q} \]

Thus in general,

\[ T^n = \begin{bmatrix} \text{abs} \ \text{nonabs} \end{bmatrix} I \frac{R}{Q^n} \]

(6)

It is well known that every object in a non-absorbing state eventually enters any of the absorbing state after a long run transition. Hence from (6), by this notion, long run transition from the 4th quadrant to the first is represented as:

\[ N = (I - Q)^{-1}, B = N \cdot R = (I - Q)^{-1} \cdot R \]

Where: \( R = d_0 \) long-run distribution to absorbing states

Figure 2 Markov transition diagram

In the method section, we showed that the long run TPM in canonical form resulted to matrix shown in equation (6).

\[ T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0111 & 0.01098 & 0.0111 & 0.8748 & 0.1022 & 0.01111 & 0.0185 & 0.0110 & 0.8681 & 0.1014 \end{bmatrix} \]

Three sets of results were obtained that were considered very significant. The first deals with the mean and variance of the level of habituations of Markov Chain among transient states.

Accordingly: Mean of habituation

(i) Mean, \( N = (I - Q)^{-1} \)

\[ \begin{bmatrix} 1.8480 & 6.8689 & 7.8107 \\ 0.8628 & 6.0391 & 7.9474 \\ 0.1561 & 6.5117 & 8.8858 \end{bmatrix} \]

The interpretation of this fundamental matrix is significant. For instance, it is evident from this fundamental matrix (N) that a contracted staff works in
The fundamental matrix, \( N \), is:

\[
N = (I - Q)^{-1} = \begin{bmatrix}
1.8480 & 66.8689 & 7.8107 \\
0.8628 & 69.0391 & 7.9474 \\
0.8561 & 67.5117 & 8.8858 \\
\end{bmatrix}
\]

The matrix \( N \) provides interesting information about the variance of the estimate of \( N = (I - Q)^{-1} \). We use the \( N \) derivatives, namely standard deviation (\( \sigma \)) to report the possible variability of the estimates of \( N \).

\[
B = N \cdot R = N \cdot d_0 = \begin{bmatrix}
0.0816 & 0.1779 & 0.0740 \\
0.0841 & 0.1624 & 0.0753 \\
0.0835 & 0.1796 & 0.0736 \\
\end{bmatrix}
\]

(i) Movement of staff within the non-absorbing states.

a. \( 6 \rightarrow 4 \) with entry 0.1561: little number of contracted staff, perhaps less than two in every ten instances, benefit from categories of leave prior to confirmation. This estimate has low standard deviation and it therefore hardly varies.

b. \( 6 \rightarrow 5 \) with entry 67.5117: for staff who are still on probation following recruitment, they undergo, on the average, six movements to different job positions as confirmed staff.

c. \( 5 \rightarrow 4 \) with entry 0.8628: This represents the number of times on the average staff may go on study leave, training leave, special leave, sabbatical leave or leave of absence. This happens perhaps about nine in every ten occasions.

d. \( 5 \rightarrow 5 \) with entry 69.0391: This variety applies to staff who remain in the system and change job positions.

e. \( 5 \rightarrow 6 \) with entry 7.9474: A confirmed staff may go back to temporary appointment, this time around, as contract staff. On the average, this may happen once before the contract may be renewed.

f. \( 5 \rightarrow 6 \) with entry 66.8689: This result suggests that staff who are on any type of the various categories of leave, on the average, return about five times to the system to undertake some assignments. For example, there are records of academic staff on leave of absence or sabbatical that come to the system to pursue their promotion documents.

g. \( 4 \rightarrow 6 \) with entry 8.8858: Staff who are newly recruited may benefit from government or institution’s scholarship in which such beneficiary proceeds on training leave and returns to the system at the expiration of the leave. Upon such return, the staff remains unconfirmed staff until he or she is due. This appears to happen only once for any staff.

(ii) The total number of movement of staff within the non-absorbing state. The matrix:

\[
\tau = \begin{bmatrix}
76.5276 \\
77.8493 \\
77.2536 \\
\end{bmatrix}
\]

reveal that generally all staff, irrespective of the starting state, undergo on the average 6 transition among the three

\[
\tau_2 = (2 \cdot N - I) \tau - \frac{\tau_{sq}}{\tau} \tau = N \cdot \xi, \quad \xi = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}
\]

\[
\tau_2 = (2 \cdot N - I) \tau - \frac{\tau_{sq}}{\tau} \tau = \begin{bmatrix}
5856.4725 \\
6060.5175 \\
5968.1179 \\
\end{bmatrix}
\]

\[
(2 \cdot N - I) \cdot \tau = \begin{bmatrix}
2.6959 & 133.7378 & 15.6215 \\
1.7256 & 137.0782 & 15.8948 \\
1.7122 & 135.0234 & 16.7716 \\
\end{bmatrix}
\]

\[
(2 \cdot N - I) \cdot \tau = \begin{bmatrix}
11824.5242 \\
12031.4359 \\
11938.1767 \\
\end{bmatrix}
\]

\[
\tau_2 = (2 \cdot N - I) \cdot \tau - \tau_{sq} = \begin{bmatrix}
5968.052 \\
5970.918 \\
5970.059 \\
\end{bmatrix}
\]

\( \tau_2 \) represents the variances associated with transition of objects among the three non-absorbing states.
non-absorbing states before being trapped in any of the absorbing states.

(iii) Transition from non-absorbing to absorbing states. Finally, 8.35% of contracted staff, 8.41% of staff stock and 8.16% of those in various categories, leave the services of the institution through normal retirement. Moreover, 17.96% each of contracted staff and those on various categories of leave, exit the organization through wastage. Furthermore, about 7.4% of contracted, 7.53% of confirmed staff and 7.36% of those on various categories of leave do leave the services of the organization through suspension. It should be pointed out that although suspension cases take protracted period to resolve, few eventually come back after long administrative and legal struggle.

CONCLUSION

Our research shows that 16.24 percent of permanent employees resign their jobs based on their own request because of health related reasons and further educational purposes and so on. Around 7.4 percent of newly recruited employees and 7.53 percent of permanent employees are fired due to disciplinary actions. Based on the findings of this research, it is recommended to take the following actions.

1. In order to retain its human resources, implement a flexible motivation policy
2. Job position planning and outlining, motivating employees by promotion
3. Improving working environment
4. Organize skill building and training sessions, regular development of proficiency and knowledge
5. Determining the cause of workforce flows and movements, adapt an optimum policy
6. Active human resources policy is deemed to be required

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